Exercises for Chapter 10

10.1 The Y data below were fit using a linear model in X. Is the model adequate? Use a Lack-of-Fit Test to determine your answer.

X	Y
-2	31
-2	33
-1	53
-1	53
0	71
0	69
1	83
1	83
2	93
2	91

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.987
R Square	0.974
Adjusted R Square	0.971
Standard Error	3.841
Observations	10

ANOVA

	df	*SS*	*MS*
Regression	**1**	**4500**	**4500**
Residual	**8**	**118**	**14.75**
Total	**9**	**4618**	

REGRESSION OUTPUT

	Coeff	*Std Error*	*t Stat*
Intercept	**66.00**	**1.214**	**54.344**
X	**15.00**	**0.859**	**17.467**

10.2 The following data were taken to study the effects of three factors on the yield of a chemical reaction. Note, the goal is to maximize the yield = 100% - % impurity.

Run	Run Order	Block	X_1	X_2	X_3	% Impurity
1	6	-1	-1	-1	-1	8.55
2	2	-1	1	-1	-1	31.33
3	4	-1	-1	1	-1	8.24
4	9	-1	1	1	-1	30.89
5	7	-1	-1	-1	1	27.79
6	5	-1	1	-1	1	29.13
7	8	-1	-1	1	1	27.37
8	1	-1	1	1	1	30.77
9	11	-1	0	0	0	19.27
10	3	-1	0	0	0	17.37
11	10	-1	0	0	0	17.76
12	14	1	-1.68	0	0	10.93
13	17	1	1.68	0	0	31.25
14	20	1	0	-1.68	0	20.74
15	18	1	0	1.68	0	19.61
16	12	1	0	0	-1.68	25.52
17	13	1	0	0	1.68	37.06
18	15	1	0	0	0	20.10
19	19	1	0	0	0	18.53
20	16	1	0	0	0	20.83

(a) Determine the best model for describing the system based on the data.

(b) Verify that any assumptions in your analysis are reasonable (via plots of residuals), and that there is no Lack-of-Fit for your model.

(c) Use that model to determine optimum operating conditions via graphical display of the equation.

(d) What is your expected yield at the optimum conditions, and what are its error limits?

10.3 Yan et al. [2011] performed a central composite experiment to determine the optimal conditions for enzymatic saccharification of food waste. The coded factors were $X_1 = ((Glucoamylaseload(U/g) - 120)/20)$, $X_2 = ((Incubationtime - 2.0)/.5)$, $X_3 = ((Temperature - 55)/5))$ and $X_4 = ((pH - 5)/.5)$, and the response was $y=$ Reducing sugar concentration (g/L). The objective was to identify conditions that would maximize the response. The design and resulting response data in standard order is shown in the table below.

(a) Fit the full quadratic model to the data.

(b) Identify the conditions that maximize the response (Reducing sugar concentration).

Run	X_1	X_2	X_3	X_4	Y
1	-1	-1	-1	-1	79.451
2	1	-1	-1	-1	119.125
3	-1	1	-1	-1	115.220
4	1	1	-1	-1	154.421
5	-1	-1	1	-1	103.253
6	1	-1	1	-1	125.526
7	-1	1	1	-1	112.332
8	1	1	1	-1	146.967
9	-1	-1	-1	1	105.214
10	1	-1	-1	1	117.396
11	-1	1	-1	1	127.318
12	1	1	-1	1	138.615
13	-1	-1	1	1	114.216
14	1	-1	1	1	128.998
15	-1	1	1	1	127.814
16	1	1	1	1	136.412
17	-2	0	0	0	96.132
18	2	0	0	0	154.218
19	0	-2	0	0	97.425
20	0	2	0	0	152.396
21	0	0	-2	0	99.229
22	0	0	2	0	114.392
23	0	0	0	-2	114.511
24	0	0	0	2	134.697
25	0	0	0	0	152.397
26	0	0	0	0	152.759
27	0	0	0	0	152.641
28	0	0	0	0	153.985
29	0	0	0	0	153.723
30	0	0	0	0	154.367

10.4 List the assumptions that are made in regression analysis, and what plot(s) you would make (if any) to check each assumption.

10.5 You fit the data from a Central-Composite design in 3 Factors with a full quadratic equation. The sum of the squared errors from the regression was:

$SSE = 175 \qquad \nu_R = 10 \qquad (s^2_R = 17.50)$

When factor X_2 was dropped from the model (4 terms), the sum of squares increased to

$SSE = 323 \qquad \nu_R = 14 \qquad (s^2_R = 23.07)$

Is X_2 needed in the model?

10.6 You took the following data to study the effects of three factors on the percent elongation of electrical tape (which is how much it stretches before it breaks). The three factors and limits are given below:

Factor	Description	Low Value	Mid Value	High Value
X_1	% Plasticizer in Formulation	-1 = 10%	0 = 25%	1 = 40%
X_2	Temperature of Compounding	-1 = 200 F	0 = 275 F	1 = 350 F
X_3	Extruder Speed	-1 = 300 rpm	0 = 450 rpm	1 = 600 rpm

Run	Run Order	X_1	X_2	X_3	Percent Elongation
1	7	-1	-1	-1	102
2	5	1	-1	-1	106
3	10	-1	1	-1	100
4	9	1	1	-1	106
5	12	-1	-1	1	115
6	3	1	-1	1	119
7	18	-1	1	1	115
8	15	1	1	1	120
9	6	-1.68	0	0	108
10	11	1.68	0	0	115
11	2	0	-1.68	0	119
12	4	0	1.68	0	117
13	13	0	0	-1.68	118
14	17	0	0	1.68	100
15	14	0	0	0	119
16	8	0	0	0	116
17	1	0	0	0	118
18	16	0	0	0	120

Analyze the results to find the best equation. Use that model to determine optimum operating conditions (maximum elongation) via graphical display of the equation. Verify that any assumptions in your analysis are reasonable (via plots of residuals), and that there is no Lack-of-Fit for your model. What is your expected response at the optimum conditions, and what are its error limits?